

Potential performance of real gas Stirling cycle heat pumps

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Heat pumps based on the reversed Stirling cycle are shown to be positively influenced by real gas effects, provided they are designed to operate in a proper region of the fluid state diagram. A simplified model of a Stirling heat pump, aimed at understanding the basic cycle thermodynamics is presented, which allows a first optimization of real gas cycles. Provided the expansion process takes place in a proper narrow region close to the critical point, efficiencies much higher than those achievable with an ideal gas and similar to those of vaporization–compression cycles are obtained. A number of zero ODP, safe fluids are considered (Xe, CHF₃, C₂F₆, CHF₃ + CF₄ mixtures) allowing optimum operation in a wide range of heat source and heat production temperatures. Only mixtures, however, are recognized to permit a fine adjustment of the fluid properties to the heat source characteristics and to the user's temperature requirements. In order to reach good energy performance, high-pressure operation (around 200 bar) and an efficient internal regeneration of heat are needed. Graphs are supplied that reveal the heat pump cycle performance for each fluid at a wide range of temperatures, pressures and cycle compression volume ratios. Loss analysis shows that fluids having a simple molecule yield the best efficiency and the minimum amount of heat regeneration. Stirling power cycles are also shown to benefit from real gas effects, with the result that at top temperatures around 400–450°C, which are probably acceptable for a number of organic fluids, a fuel to work conversion efficiency around 25–30% seems possible for a cogenerative prime mover. The performance of such motors, intended for heat pump drives, are given for C₂HF₅ and C₃F₈ fluids. Very high pressures are required to optimize the cycle performance. Preliminary information on the prospective characteristics of a fuel powered Stirling–Stirling low-grade heat generator is given. Copyright © 1996 Elsevier Science Ltd and IIR

(Keywords: heat transformer; thermodynamic cycle; Stirling; real gas; performance; engine; heat pump; cogeneration)

Performance potentielle de pompes à chaleur à cycle de Stirling et gaz réel

On considère que les pompes à chaleur basées sur le cycle inversé de Stirling sont influencées de façon positive par les effets des gaz réels, à condition qu'elles soient conçues pour fonctionner dans une région appropriée du diagramme de l'état des fluides. On présente un modèle simplifié d'une pompe à chaleur à cycle de Stirling, dont le but est de comprendre la thermodynamique fondamentale du cycle, ce qui permettrait une première optimisation des cycles de gaz réel. Si la phase de détente se produit dans une région étroite adéquate, près du point critique, on obtient des efficacités bien supérieures à celles que l'on peut obtenir avec un gaz idéal et similaires à celles des cycles à évaporation–compression. On considère plusieurs fluides sûrs, ayant un potentiel de destruction de l'ozone nul (Xe, CHF₃, C₂F₆, mélanges CHF₃ + CH₄), permettant un fonctionnement optimal dans une large plage des températures de la source de chaleur et de la production de chaleur. Cependant, on ne retient que les fluides permettant un ajustement étroit entre propriétés du fluide et caractéristiques de la source de chaleur et exigences de température de l'utilisateur. Si l'on veut atteindre une bonne performance énergétique, un fonctionnement à haute pression (autour de 200 bars) et une phase de régénération de chaleur efficace sont nécessaires. On présente des graphiques qui révèlent la performance du cycle de la pompe à chaleur pour chaque fluide, dans de larges plages de températures, pressions et rapports volumiques du cycle de compression. L'analyse des pertes montre que les fluides à molécule simple présentent la meilleure efficacité et donnent la quantité minimale de chaleur de régénération. On montre également que les cycles de Stirling tirent des avantages des effets de gaz réels, et qu'à des températures élevées (autour de 400–500°C), qui sont probablement acceptables pour un certain nombre de fluides, une efficacité de conversion chaleur en travail autour de 25 à 30% semble possible, d'où possible utilisation du couple moteur Stirling–pompe à chaleur Stirling à gaz réel, en cogénération. La performance de ces moteurs Stirling, servant à actionner des pompes à chaleur, est donnée pour les fluides C₂HF₅ et C₃F₈. Des pressions très élevées sont nécessaires pour optimiser la performance du cycle. On donne quelques renseignements préliminaires sur des caractéristiques d'un générateur de chaleur de moindre qualité, à cycle Stirling–Stirling, actionné par un combustible. Copyright © 1996 Elsevier Science Ltd and IIR

(Mots-clés: transformateur de chaleur; cycle thermodynamique; Stirling; Gaz réel; performance; moteur; pompe à chaleur; cogénération)

Nomenclature

COP	Heating coefficient of performance (the ratio of the cycle heat output to the net work consumption)	v	Specific volume ($\text{m}^3 \text{kg}^{-1}$)
COP*	Cooling coefficient of performance (the ratio of the low grade heat input to the work consumption) (Equation 4)	W_c	Compression work (J kg^{-1})
COP* _{id}	Ideal cooling coefficient of performance evaluated between the thermodynamic equivalent temperatures T_{eqs} and T_{equ} defined in Equation (3)	W_e	Expansion work (J kg^{-1})
C_v^0	Molar specific heat at constant volume ($\text{J mol}^{-1} \text{kg}^{-1}$)	W_n	Net work ($= W_c - W_e$) (J kg^{-1})
p	Pressure (bar)	W_c'	Ideal compression work (J kg^{-1})
p_{max}	Maximum cycle pressure (bar)	W_e'	Ideal expansion work (J kg^{-1})
p_{min}	Minimum cycle pressure (bar)	z	Compressibility factor ($= p v / R T$)
Q_s	Heat absorbed from source (J kg^{-1})	Greek letters	
Q_u	Heat delivered to user (J kg^{-1})	Δt_{min}	Regenerator minimum temperature difference for both direct and reversed cycles ($^{\circ}\text{C}$)
QF	Cycle thermodynamic quality factor (the ratio of actual to ideal COP*) (Equation 5)	η_c	Compression efficiency ($= W_c' / W_c$)
R	Universal gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)	η_e	Expansion efficiency ($= W_e' / W_e$)
s	Specific entropy ($\text{J kg}^{-1} \text{K}^{-1}$)	ρ	Volume compression and expansion ratio $v_1/v_2 = v_4/v_5$ in the heat pump cycles; $v_1/v_7 = v_8/v_3$ in power cycles) (Figures 1b and 10b)
T	Absolute temperature (K)	τ	Maximum to minimum cycle temperature ratio (T_2/T_4 in heat pump cycles; T_3/T_1 in power cycles) (Figures 1b and 10b)
t	Temperature ($^{\circ}\text{C}$)	Subscripts	
T_{eqs}	Equivalent thermodynamic source temperature (defined in Equation 2) (K)	1–8	Points of heat pump and of power cycles (Figures 1b and 10b)
T_{equ}	Equivalent thermodynamic user temperature (defined in Equation 1) (K)	cr	Critical conditions
u	Specific internal energy (J kg^{-1})	is	Isentropic process
		r	Reduced conditions ($p_r = p/p_{cr}$, $T_r = T/T_{cr}$)

In a recent paper¹ the thermodynamic merits of reversed Brayton cycles, in which the expansion takes place in the critical region of the working fluid, were examined. The main shortcomings of the proposed cycles turned out to be the high pressures involved (two to four times the fluid critical pressure) and the large amount of internal regeneration which is required to assure good energy effectiveness. Positive displacement engines, as developed for performing either Stirling or Ericsson cycles (both direct and reversed) have the potential of handling high-pressure fluids without problems and of assuring an efficient regeneration heat transfer using an economic, compact heat-storing matrix.

Although both cycles feature, ideally, isothermal compression and expansion it is known that their basic character is preserved when the two isothermal processes are replaced by isentropes, which represent a better approximation for the expansion and compression performed in actual engines². Consequently an Ericsson engine could operate approximately according to the thermodynamic cycles illustrated in Ref. 1 at the moderate power levels which are typical of positive displacement devices. Stirling cycle engines, which do not require the use of valves, represent a simpler solution and demonstrated high efficiency and reliability when operated as low-temperature refrigerators, mainly for military use.

Although the thermodynamics of the reversed Stirling cycle is somewhat different from that of the reversed Ericsson or of the reversed Brayton cycle, mainly because of the isochoric pressure changes which adds

to those produced by mechanical means, real gas effects are likely to play a positive role in this case also.

The aim of this paper is to extend the evaluation of real gas effects to Stirling-cycle-based heat pumps and refrigerators. If the high efficiency prospects for this varied version of the Stirling heat pump or refrigerator could be practically demonstrated, a new tool would be available for general use having the potential of alleviating the ambient impact of cold and heat generation based on mechanical means.

Basic thermodynamics

As an explanatory example, an ideal Stirling heat pump cycle using CHF₃ (HFC-23) as working fluid is represented on the right-hand side of Figures 1a and 1b, assuming an ideal gas behaviour on the pressure–volume and on the temperature–entropy diagram (cycle I, 1–2–3–4). With reference to Figure 1c, starting from point 1 (all fluid in the hot compression space at its maximum volume and temperature), the compression piston at right moves leftward to point 2, reducing the volume to a fraction of its starting value, while the temperature is kept constant by transferring heat to the user. Expansion and compression pistons then move simultaneously leftward: the working fluid emerges into the expansion space after having been cooled to point 3 (minimum temperature) by the regenerator porous matrix. The expansion piston moves further leftward to point 4, while temperature is kept constant by introducing low-grade heat from the surroundings. Now both pistons

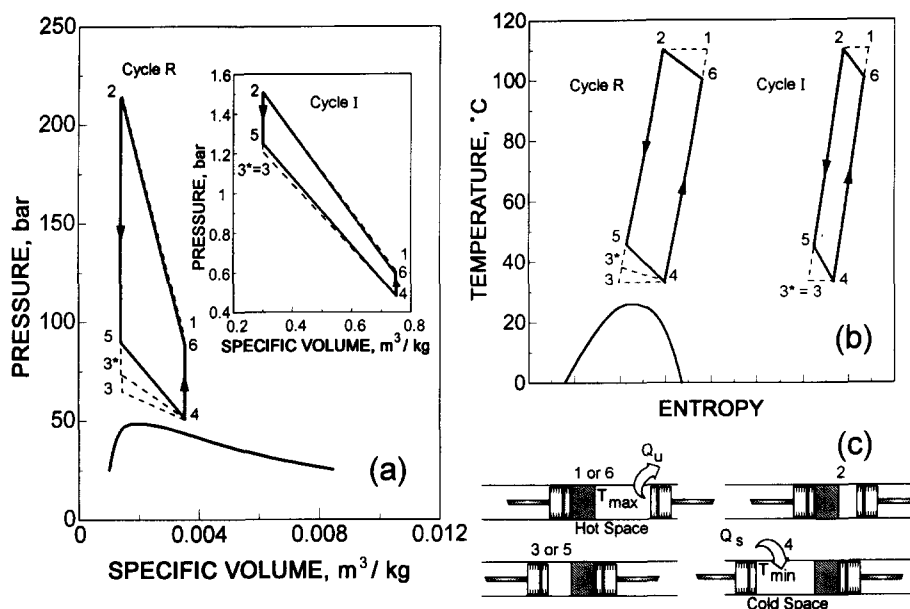


Figure 1 Ideal and real gas Stirling heat pump cycles in the pressure–volume (a) and in the temperature–entropy (b) diagram for (trifluoromethane). Conceptual engine operation (c)

Figure 1 Cycles idéal et réel d'une pompe à chaleur de Stirling dans le diagramme pression–volume (a) et température–entropie (b) pour le trifluorométhane. Fonctionnement du moteur (c)

move simultaneously rightwards causing the fluid to absorb heat from the regenerator at constant volume, thus emerging in the compression space at the maximum temperature and closing the cycle.

The compression ratio $\rho = v_{\max}/v_{\min}$ is 2.5 and the temperature ratio $\tau = T_{\max}/T_{\min}$ is 1.25. Useful heat Q_u is equal to compression work while the ratio of compression (W_c') to expansion work (W_e') coincide with temperature ratio τ , which means that, in this example, for each unit of heat generated an almost double amount of combined compression–expansion work has been done by the pistons.

In cycle R (1–2–3*–4) minimum pressure and temperature, p_4 and T_4 , are slightly above critical ($p_{r4} = 1.05$, $T_{r4} = 1.025$) and the fluid exhibits strong real gas effects. The higher isochoric heat capacity of the compressed fluid prevents a through (reversible) regeneration of the heat available between $T_{\max}$ and $T_{\min}$ even for an infinite heat transfer surface: $u_2 - u_3 > u_1 - u_4$; $Q_R = u_2 - u_3^* = u_1 - u_4$. Thermally driven, isochoric pressure changes, which are very moderate in cycle I ($p_1/p_4 = p_2/p_3 = 1.25$), are boosted in cycle R by the strong variation with temperature of the compressibility factor, mainly along the high density isochor ($p_2/p_3^* = (T_2/T_3^*)(z_2/z_3^*) = 3.24$; $p_1/p_4 = (T_1/T_4)(z_1/z_4) = 1.82$) with the result that expansion work, dealing with particularly low pressures, becomes much smaller than compression work and the ratio of the combined piston work to the useful heat output is much smaller than in the ideal gas case ($(W_c' + W_e')/Q_u = 0.52$ instead of 1.8).

Assuming reversible work exchange cycle I is strictly ideal (equivalent to a reversed Carnot cycle). Cycle R is only slightly less efficient due to the not fully reversible transfer of the regenerated heat. The performance of such idealized cycles is not a meaningful index of the actual merits of the cycle configurations considered. A thorough analysis of the losses of a Stirling engine, whose complex nature is largely due to its unsteady

operation, is beyond the scope of this paper. A simplified method was developed which takes into account the main reasons of losses which are typical of Stirling engines. With reference to both cycles I and R the following non-idealities were introduced: (1) a finite minimum temperature difference $\Delta t_{\min}$ was assumed for the regeneration process $\Delta t_{\min} = t_2 - t_6 = t_5 - t_4$ in cycle I; $\Delta t_{\min} = t_2 - t_6$ in cycle R. Owing to the higher C_v of the compressed fluid $t_5 - t_4 > t_2 - t_6$, i.e. in all calculations $\Delta t_{\min}$ was always found at the regenerator end. Furthermore, notwithstanding the peculiar thermodynamics of the transcritical region, the temperature–internal energy profile within the regenerator was found to be basically linear, isochoric heat capacity exhibiting a rather regular behaviour in contrast with what happens for heat capacity at constant pressure. The influence of transcritical transport properties on the actual regenerator performance is beyond the scope of this paper; (2) compression is performed at a slightly increasing temperature ($t_2 > t_6$) and expansion at slightly decreasing temperature ($t_4 < t_5$), consistently with the regenerator performance; (3) assuming a fluid volumetric behaviour of the type $p v^m = \text{const.}$ in which m is an appropriate exponent, in both compression and expansion the actual work exchange is computed from the ideal (reversible) work by introducing a compression and an expansion efficiency as follows: $W_c' = -\int p dv$; $W_e' = \int p dv$; $W_c = W_c'/\eta_c$; $W_e = W_e'/\eta_e$. Pressure losses, which are known to be important in Stirling engines, diminishing expansion and increasing compression work³ were neglected. Their effect is believed to be adequately summarized by η_c and by η_e which become the formal tool for taking into account, on an energy equivalent basis, all kinds of aerodynamic losses (including windage and piston leakage losses). Mechanical losses are not taken into consideration.

The above cycle will be conventionally referred to as a 'real' cycle. To compare its performance with that of an

ideal system, energy equivalent user's and source temperatures are defined as follows:

$$T_{\text{eq u}} = \left(\int_6^2 p \, dv + (u_2 - u_6) \right) / (s_2 - s_6) \quad (1)$$

$$T_{\text{eq s}} = \left(\int_5^4 p \, dv + (u_4 - u_5) \right) / (s_4 - s_5) \quad (2)$$

The exergy of the heat produced between T_6 and T_2 is equal to the exergy of the same amount of heat at $T_{\text{eq u}}$. Similarly, the exergy of the heat absorbed between T_5 and T_4 is equal to the exergy at $T_{\text{eq s}}$. Observing that the true merit of the heat pump is the absorption and upgrading of low temperature heat, an ideal and an actual coefficient of performance are computed as follows:

$$\text{COP}_{\text{id}}^* = \frac{Q_s'}{W_n'} = \frac{T_{\text{eq s}}}{T_{\text{eq u}} - T_{\text{eq s}}} \quad (3)$$

$$\text{COP}^* = \frac{Q_s}{W_n} \quad (4)$$

in which W_n' and W_n are the net work consumptions in the ideal and real cycle. A quality factor QF , having the meaning of a second law efficiency is then calculated:

$$QF = \frac{\text{COP}^*}{\text{COP}_{\text{id}}^*} \quad (5)$$

and is believed to summarize the energy merits of the heat pump cycle. Assuming $\eta_c = \eta_e = 0.85$, $\Delta t_{\text{min}} = 10^\circ\text{C}$, $\rho = 2.5$ and $\tau = 1.25$, ideal gas–real cycle I (6–2–5–4, Figure 1b) lifts heat from $T_{\text{eq s}} = 312.3\text{ K}$ to $T_{\text{eq u}} = 378.2\text{ K}$ with a COP^* of 1.12 and a QF of 0.236; real gas–real cycle R (6–2–5–4) lifts heat from $T_{\text{eq s}} = 313.6\text{ K}$ to $T_{\text{eq u}} = 379.6\text{ K}$ with a COP^* of 2.58 and a QF of 0.542. $(W_c + W_e)/Q_u$ is 1.86 in the first case and 0.6 in the second case, which highlights the basic reason for the better performance of real gas cycles. Thermal pressure changes brought about by the regeneration process enhanced by the real gas effects, make the expansion work a small fraction of the compression work (36% against 60% for the ideal gas cycles), thus reducing overall piston works and related fluid-dynamic losses which, in our assumptions, are assumed to be a fixed fraction of work exchanges. Alternately the benefits of the real gas effects can be explained as follows. In both cycle I and R user's heat is computed from compression work and internal energy change:

$$Q_u = W_c - (u_2 - u_6) \quad (6)$$

Since $t_2 > t_6$ in cycle I, $u_2 - u_6$ is a small, positive amount (about 14% of W_c) and reduces the heat delivery. In cycle R, on the other hand, the much stronger molecular interaction at point 2 makes $u_2 - u_6$ negative and substantial (about 128% of W_c) with the result that the heat delivery becomes large in comparison to compression work. In our example $u_2 - u_6$ is -39.5 kJ kg^{-1} , which has a thermodynamic meaning not far from that of the heat of condensation in conventional compression cycles. The low molecular energy level reached by the fluid at the end of the heat delivery process, makes available an amount of heat much larger than that corresponding to the compression work input.

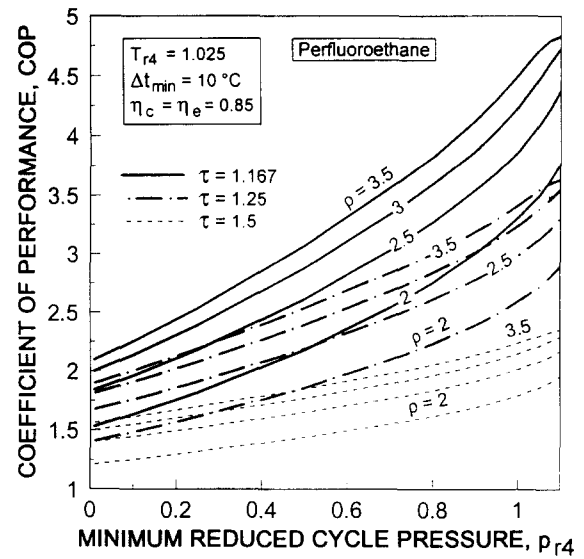


Figure 2 Influence of minimum cycle pressure on COP for perfluoroethane

Figure 2 Influence de la pression minimale du cycle sur le COP pour le perfluoroéthane

Heat pump cycle performance

Variables

With reference to Figure 1, the main variables which allow the shaping of cycle R are: (1) the position of point 4, at the minimum pressure and temperature, with respect to the saturation curve (implying the selection of T_{r4} and p_{r4}); (2) the volume compression ratio ρ ; (3) the temperature ratio $\tau = T_{\text{max}}/T_{\text{min}} = T_2/T_4$ which controls the temperature raising of the low grade heat; (4) the quality of the engine, summarized in our simplified model by η_c , η_e and Δt_{min} ; (5) the nature of the working fluid which can be summarized by a single parameter: the molar specific heat in the ideal gas state.

For given temperature requirements, the basic optimization parameters are T_{r4} , p_{r4} and ρ . Extensive computation showed that optimum T_4 is always slightly above T_{cr} (e.g. $T_{r4} = 1.01 - 1.03$). At higher T_r the real gas effects quickly fade, while at lower T_r the cycle could enter the two phase or the liquid region in the latter case with an intolerable pressure rise under the constant volume heating).

The influence of p_{r4} on cycle performance is illustrated, for C_2F_6 , in Figure 2 giving the cycle COP for various τ and ρ as a function of p_{r4} at $T_{r4} = 1.025$. For a given T_4 and τ heat is absorbed and delivered approximately at fixed temperatures[†] and the variation of COP reflects a similar variation of the cycle thermodynamic quality. The positive influence of real gas effects displays itself gradually from $p_{r4} = 0$ to 1.1, or even beyond, with the largest benefits (COP more than doubled) obtained at the lowest τ , for which the ideal gas expansion work is particularly large in comparison with compression work.

[†] At the cold end of the regenerator a Δt somewhat larger than the selected minimum slightly affects the best absorption temperature in real gas cycles

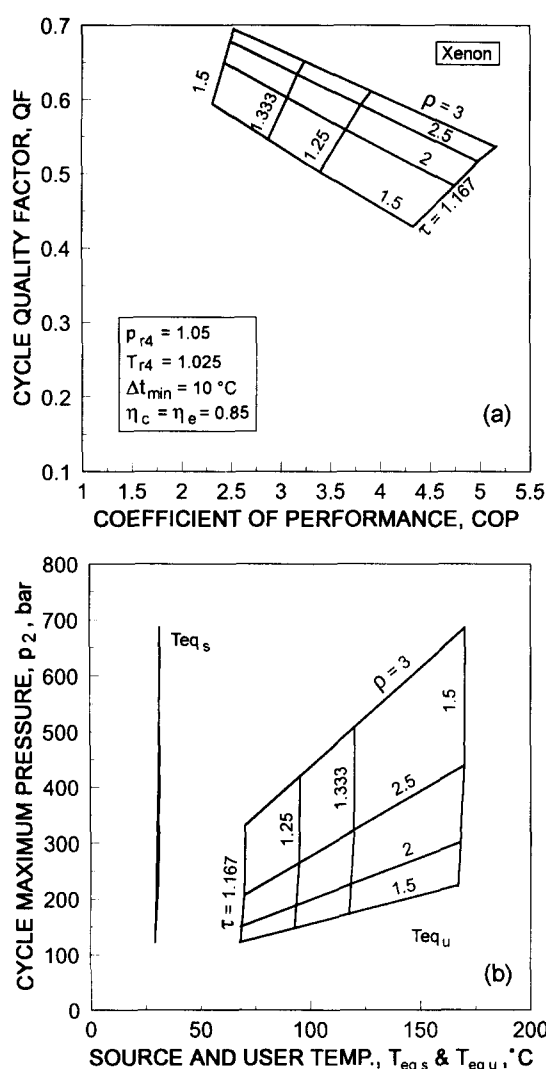


Figure 3 Real gas Xe heat pump cycles: energy performance (a), heat source and heat output temperatures (b)

Figure 3 Cycles d'une pompe à chaleur avec du xénon (gaz réel): performance énergétique (a), températures de la source de chaleur et de la sortie de chaleur (b)

Working fluid evaluation

In the cycle considered, heat absorption temperatures are slightly higher than the fluid critical temperature. Hence a rather strict connection exists between the heat source temperature and the working fluid nature. A number of potentially useful fluids Xe, CHF_3 (HFC23), C_2F_6 (FC116) and CO_2 were examined in detail having critical temperatures between 16.5°C (Xe) and 30.9°C (CO_2). All fluids are non-toxic, non-flammable and harmless to stratospheric ozone. Lower critical temperatures which would be needed to use sources around or below ambient temperature could be obtained by mixing two appropriate fluids. For example, a mixture with the following molar composition: 42.7% CF_4 (R14, $t_{cr} = -45.5^\circ\text{C}$), 57.3% CHF_3 (HFC23, $t_{cr} = 25.6^\circ\text{C}$) has a critical temperature of -10°C and is adequate for sources at almost 0°C .

The following assumptions were made for all fluids: $\eta_c = \eta_e = 0.85$; $\Delta t_{min} = 10^\circ\text{C}$, $p_{r4} = 1.05$, $T_{r4} = 1.025$, τ was varied between 1.167 and 1.5, ρ between 1.5 and 3. with reference to Figures 3–6 for each fluid, a first graph reports QF as a function of $\text{COP} = Q_u/W_n$, while a second graph gives the cycle maximum pressure p_2 and

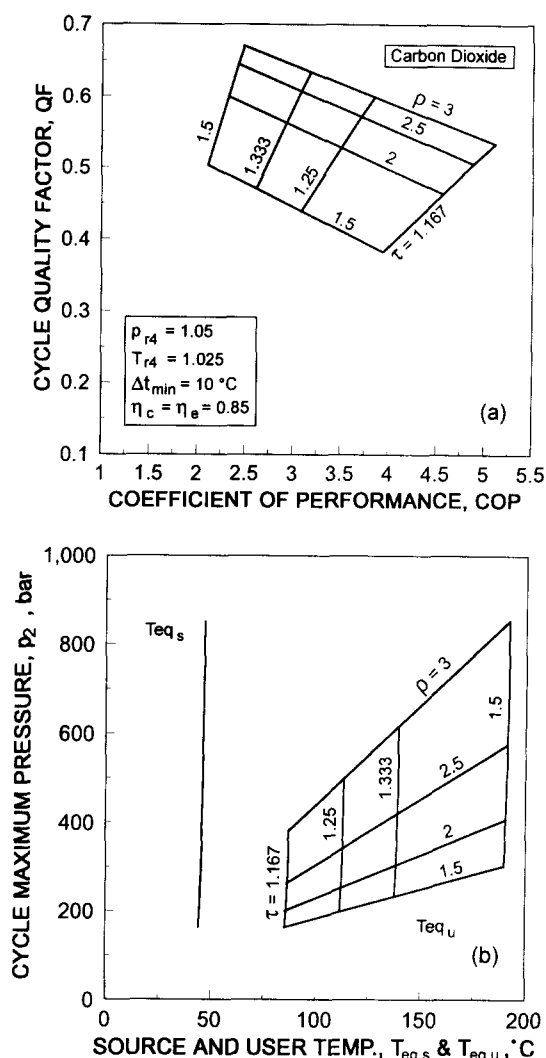


Figure 4 Real gas carbon dioxide heat pump cycles: energy performance (a), heat source and heat output temperatures (b)

Figure 4 Cycles d'une pompe à chaleur au dioxyde de carbone (gaz réel): performance énergétique (a), températures de la source de chaleur et de la sortie de chaleur (b)

the heat absorption and heat rejection temperatures as energy equivalent average. Large ρ and large τ have both a positive influence on QF . The best thermodynamic performance, however, is associated with comparatively low COP (below 2.5). QF above 0.5 are achievable with all fluids, while Xe and CO_2 exhibit a small operational region where $QF > 0.6$. This range of performance is similar to that of small scale conventional vaporization–compression cycles at comparatively large τ . The combined thermal–mechanical compression starting from an already high pressure level produces very high top pressures: only a careful selection of the operating parameters can limit p_{max} to 200–250 bar, still preserving a good cycle effectiveness.

For example, Xe at $p_{min} = 61$ bar, $p_{max} = 213$ bar, $\rho = 2$ produces heat at $t_{equ} = 118^\circ\text{C}$ from $t_{eqs} = 30^\circ\text{C}$ with a COP of 3.08 (69% of COP_{id}) and a QF of 0.60; CO_2 at $p_{min} = 77$ bar, $p_{max} = 269$ bar, $\rho = 2$ produces heat at $t_{equ} = 128^\circ\text{C}$ from $t_{eqs} = 45^\circ\text{C}$ with a COP of 3.12 (64% of COP_{id}) and a QF of 0.55; CHF_3 at $p_{min} = 51$ bar, $p_{max} = 243$ bar, $\rho = 2.5$ produces heat at $t_{equ} = 122^\circ\text{C}$ from $t_{eqs} = 41^\circ\text{C}$ with a COP of 3.17 (65% of COP_{id}) and QF

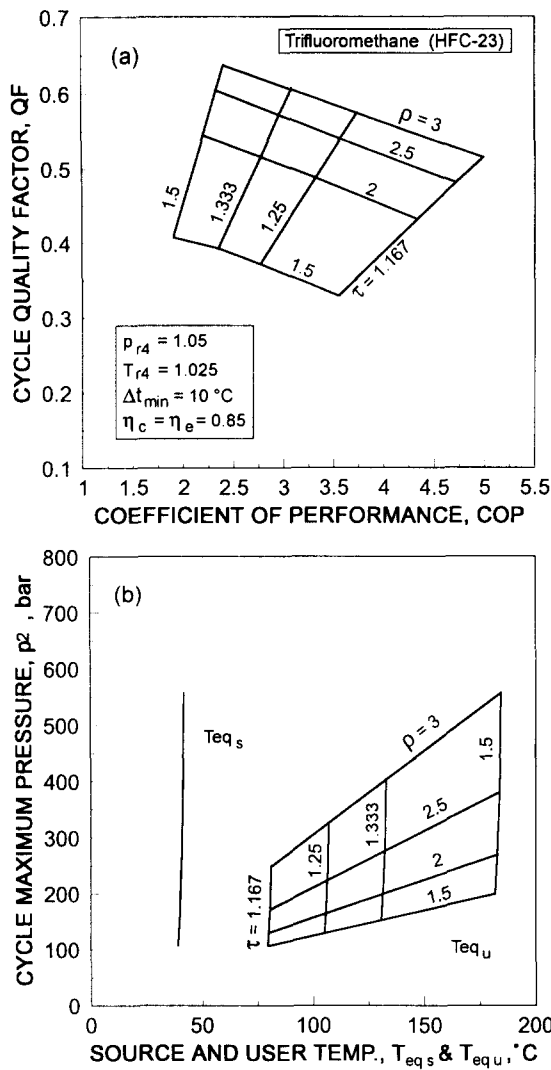


Figure 5 Real gas trifluoromethane heat pump cycles: energy performance (a), heat source and heat output temperatures (b)

Figure 5 Cycles d'une pompe à chaleur au trifluorométhane (gaz réel): performance énergétique (a), températures de la source de chaleur et de la sortie de chaleur (b)

of 0.56. C_2F_6 having a more complex molecular structure requires large ρ to attain a similar efficiency: at $p_{min} = 32$ bar, $p_{max} = 224$ bar, $\rho = 3$, C_2F_6 produces heat at $t_{equ} = 115^\circ\text{C}$ from $t_{eqs} = 35^\circ\text{C}$ with a COP of 3.00 (62% of COP_{id}) and a QF of 0.52. Since temperature variation in both heat absorption and rejection is around 10°C , the average temperatures reported are believed to give adequate information for most applications.

Loss analysis

Neglecting losses due to the irreversible transfer of heat from the source, to the user the lower than ideal cycle performance is due only to internal losses, i.e. compressor, expander and regenerator entropy production. The increase in work consumption per unit heat output with respect to the theoretical minimum of the same T_{equ} and T_{eqs} brought about by the various irreversible processes is reported in Figure 7 as a function of ρ for Xe and C_2F_6 at fixed T_{r4} , p_{r4} and τ .

In the whole range of variation of ρ , C_2F_6 exhibits a comparatively large regenerator loss which contrasts

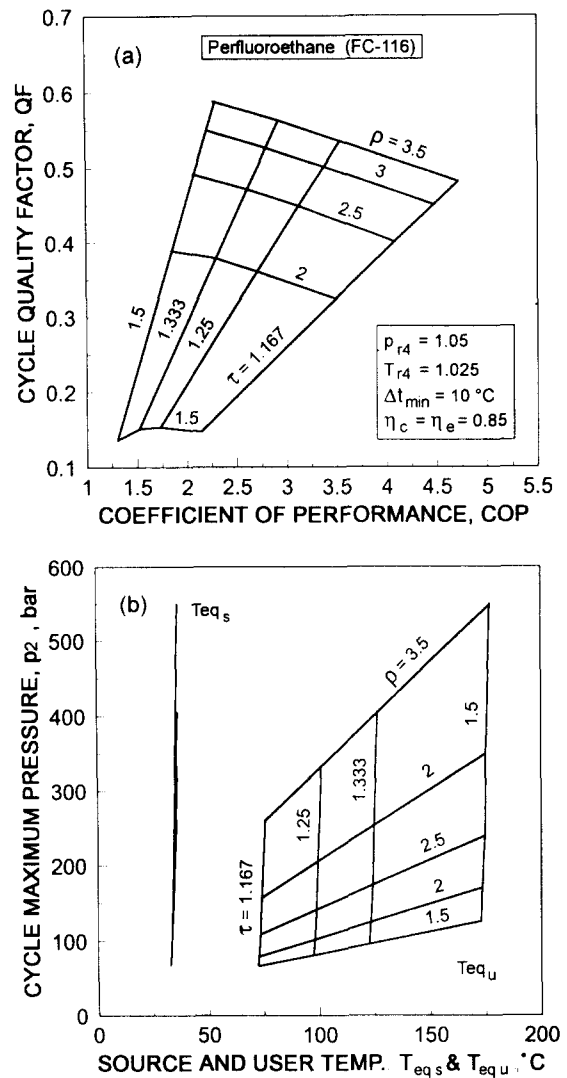


Figure 6 Real gas perfluoroethane heat pump cycles: energy performance (a), heat source and heat output temperatures (b)

Figure 6 Cycles d'une pompe à chaleur au perfluoroéthane (gaz réel): performance énergétique (a), températures de la source de chaleur et de la sortie de chaleur (b)

the almost negligible loss in the Xe regenerator. At the smallest ρ , compressor and expander losses are also more severe in C_2F_6 cycles. The reason for this is clear from the diagrams of Figure 8, giving the detailed flow of energy in a C_2F_6 and in a Xe cycle with $\rho = 2.5$, $\tau = 1.25$. Owing to the complex molecular structure of C_2F_6 , the amount of heat regenerated is 1.8 times the heat output, while for Xe Q_R is only $0.28 Q_u$. In molar terms both fluids exhibit a strict similarity in p - v behaviour and work exchanges, while the heat regenerated is roughly proportional to the molar C_v^0 , which for C_2F_6 is some six times larger than for Xe. Such a different thermal behaviour is also indirectly responsible for the comparatively large compressor and expander losses in C_2F_6 cycles at low ρ . For $\rho = 1.5$, for example, in molar terms compression and expansion works and related fluid-dynamic losses are similar for both fluids while the molar heat output $\bar{Q}_u = \bar{W}_c + (\bar{u}_6 - \bar{u}_2)$ is much smaller for C_2F_6 for which $(\bar{u}_6 - \bar{u}_2)$ approaches zero. This happens because the comparatively low value of $\bar{u}_2$, which is due to a moderate real gas effect (small ρ), is offset by a similarly low value of $\bar{u}_6$ brought about by a t_6 lower by some 10°C than t_2

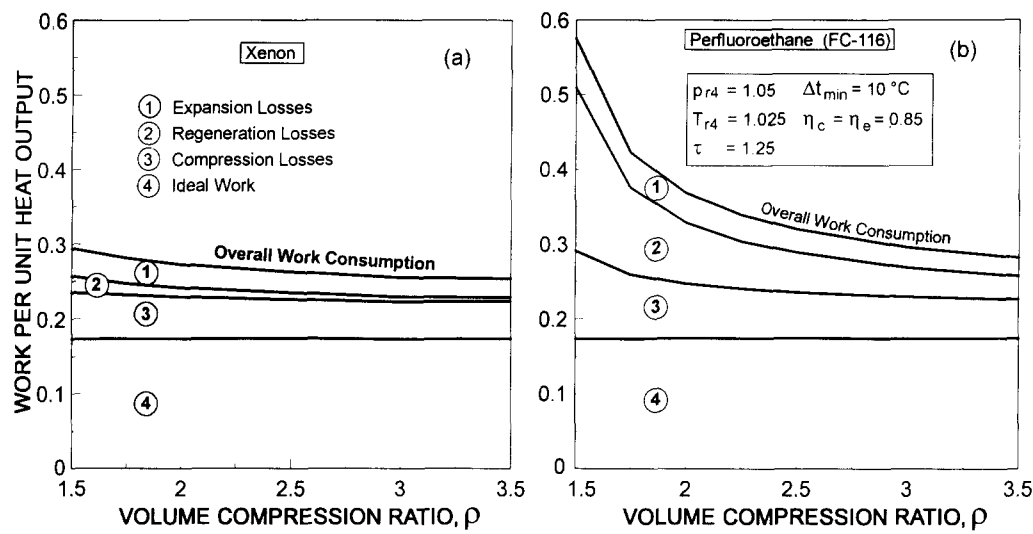


Figure 7 Incremental work consumption over ideal due to the various irreversible processes, for Xe and for perfluoroethane heat pump cycles. Calculation assumptions for both fluids are given in the figure

Figure 7 *Accroissement de la quantité de travail par rapport à l'idéal, dû aux différents processus irréversibles, pour des cycles de pompes à chaleur au xénon et au perfluoroéthane. Les hypothèses de calcul pour les deux fluides sont donnés dans la figure*

and by the large molar C_v^o of the fluid. As a result, fluid-dynamic losses per unit heat output become particularly severe. A more detailed report of the negative influence of C_v^o on COP at various τ is given in Figure 9. At $\tau = 1.25$, COP drops from 3.8 to 3.0 changing from $C_v^o = 12.5 \text{ J mol}^{-1} \text{ K}^{-1}$ (Xe at $T_r = 0.7$) to $C_v^o = 124.5 \text{ J mol}^{-1} \text{ K}^{-1}$ (C_3F_8 at $T_r = 0.7$); a similar performance loss is experienced also for other values of τ .

Thermally driven Stirling–Stirling heat pump

A Stirling engine could be used to drive a Stirling cycle heat pump. It could benefit from real gas effects for the

same basic thermodynamic reasons as heat pump cycles do. In this case compression work rather than expansion work is exceptionally small with benefits on fluid-dynamic losses. Provided the fluid critical temperature is properly selected, waste heat is rejected at a useful temperature and added to the heat pump output. Since, in optimized power cycles heat rejection mean temperature turns out to be slightly above T_{cr} , a fluid with a critical temperature slightly below the temperature requirements of the user should be used, while for the heat pump cycle a fluid with T_{cr} slightly below the heat source temperature is needed. Consequently the same fluid cannot properly perform both tasks.

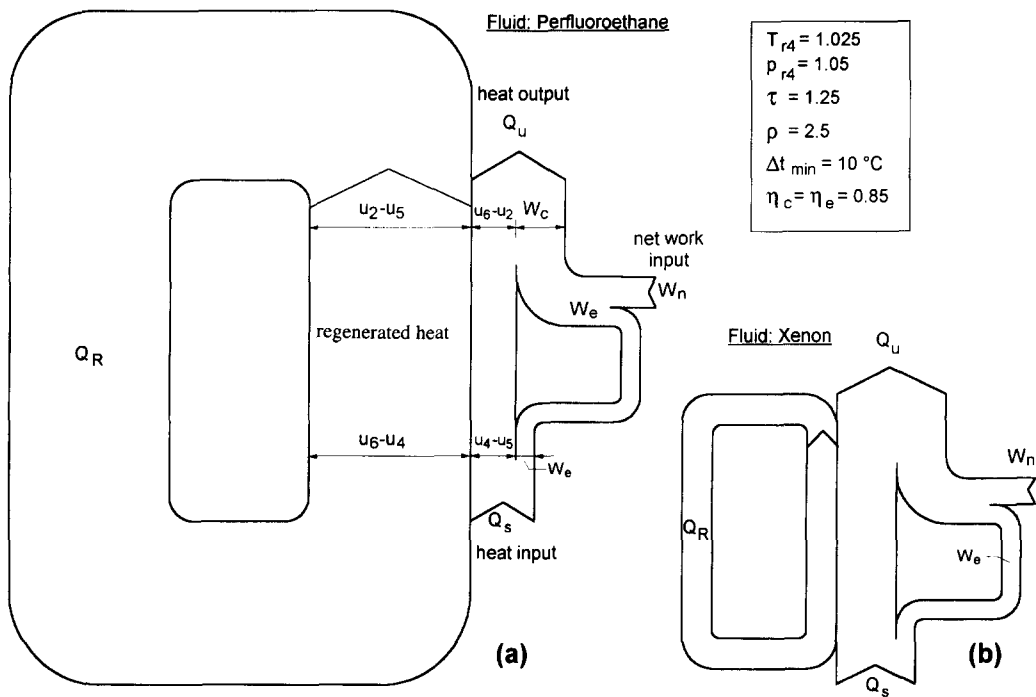


Figure 8 Energy flow in perfluoroethane and Xenon real gas heat pump cycles

Figure 8 *Ecoulement énergétique dans des cycles de pompes à chaleur au perfluoroéthane et au xénon (gaz réels)*

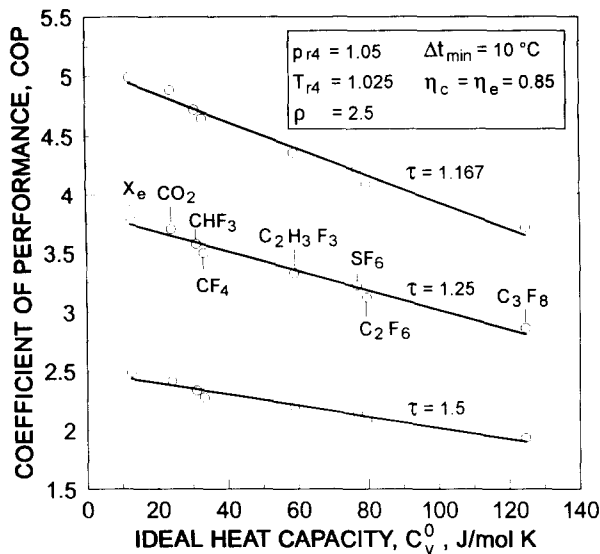


Figure 9 Influence of ideal gas molar specific heat on energy performance of real gas heat pump cycles

Figure 9 Influence de la chaleur spécifique molaire d'un gaz idéal sur la performance énergétique de cycles de pompes à chaleur (gaz réel)

Furthermore, the rather strict link between T_{cr} and the temperature of the heat produced implies, for a practical use, the availability of fluids with critical temperatures in well-defined ranges, while working media with an adjustable T_{cr} would be even more desirable. Only organic fluids or mixtures of them have the potential of meeting similar requirements. The limited thermal stability of organic molecules calls for a comparatively modest top temperature, in the range 400–450°C, at which a cycle efficiency around 25–35% should be obtained if the prime mover has to be competitive with existing motors. A simplified program was developed to evaluate the basic performance of real gas Stirling power cycles. The ideal Stirling cycle of Figure 10a (isothermal compression and expansion, reversible regeneration) was modified in order to take into account, in a schematic form, fluid-dynamic and heat-transfer losses.

The working fluid, after having been pre-cooled to the cycle minimum temperature t_1 (Figure 10b), is compressed at increasing temperatures to t_7 with an efficiency η_c with respect to a reversible process. Temperature t_7 is selected halfway between $t_2 = t_1$ and t_{7is} , which would be reached at the end of an isentropic compression. Similarly during the expansion from point 3, the fluid is allowed to cool to t_8 halfway between t_3 and the isentropic final temperature t_{8is} , while delivering external work with an efficiency η_e with respect to a reversible process. Regenerated heat is introduced in the compressed fluid at constant volume from t_7 to a temperature t_6 such that the minimum temperature difference in the regenerator, wherever it occurs, is equal to a selected input datum Δt_{min} . External heat is used to pre-heat the fluid from t_6 to t_3 and to limit the temperature drop during the expansion.

The working fluid is finally cooled at constant volume firstly to t_5 within the regenerator and then to t_1 in the pre-cooler.

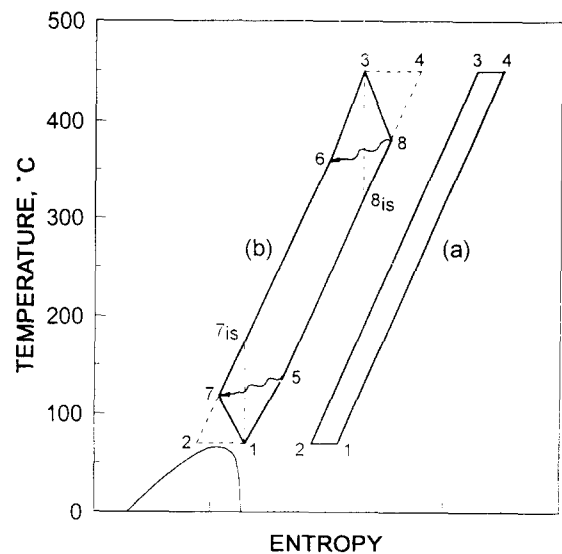


Figure 10 Moderate temperature cogenerative Stirling power cycles: ideal gas, reversible cycle (a) and real gas cycle (b)

Figure 10 Cycles de Stirling de cogénération à température modérée: gaz idéal, cycle réversible (a) et cycle de gaz réel (b)

Two fluids having a good thermal stability⁵⁻⁷ and a suitable critical temperature were considered for a detailed evaluation: C_2HF_5 (HFC125, $t_{cr} = 66.25^\circ\text{C}$) and C_3F_8 (FC218, $t_{cr} = 71.95^\circ\text{C}$). Even if a definite acceptable top temperature is not known for either fluid, experimental tests support the conclusion that an operation maximum temperature of 400–450°C represents a reasonable goal.

Assuming $T_{1r} = 1.01$, $\eta_c = \eta_e = 0.85$, $\Delta t_{min} = 20^\circ\text{C}$, the cycle performance was optimized varying p_{1r} and ρ , as shown in Figures 11a and 11b, in which the cycle efficiency (the ratio of work output to the heat input) is reported as a function of the top pressure p_3 for $p_{1r} = 0.6$ and 0.9 and for $t_3 = 400$ and 450°C.

Limiting the top pressure to 250–300 bar, which should represent an acceptable level owing to the mechanical conception of the Stirling engine, and to the moderate top temperature, an efficiency in the range 26–31% could be achieved. Owing to the simplified computational model, these figures should be considered, in comparison with the efficiencies of perfect gas cycles obtained with the same model and the same assumptions. For example, C_2HF_5 at $t_{min} = 71^\circ\text{C}$, $t_{max} = 400^\circ\text{C}$, $p_{min} = 32$ bar, $p_{max} = 270$ bar, $\rho = 2.3$ exhibits a computed efficiency of 27.4%. Assuming an ideal gas behaviour, at the same limiting temperatures and ρ , the cycle efficiency would be 16.7%. The good performance of the real gas cycle is basically due to the small compression work (less than half that of the ideal gas) which is detracted from a substantially fixed expansion work. The more than twofold net work output not only guarantees a high efficiency for the real gas cycle, but also halves the regenerator duty per unit power output. Similarly C_3F_8 at $t_{min} = 75.5^\circ\text{C}$, $t_{max} = 400^\circ\text{C}$, $p_{min} = 24$ bar, $p_{max} = 279$ bar, $\rho = 3.0$ exhibits a computed efficiency of 27.0% (15.1% in the ideal gas assumption).

For practical use, the mean waste heat rejection temperature (as an energy equivalent) is more meaningful than the minimum cycle temperature, and for a given T_{1r}

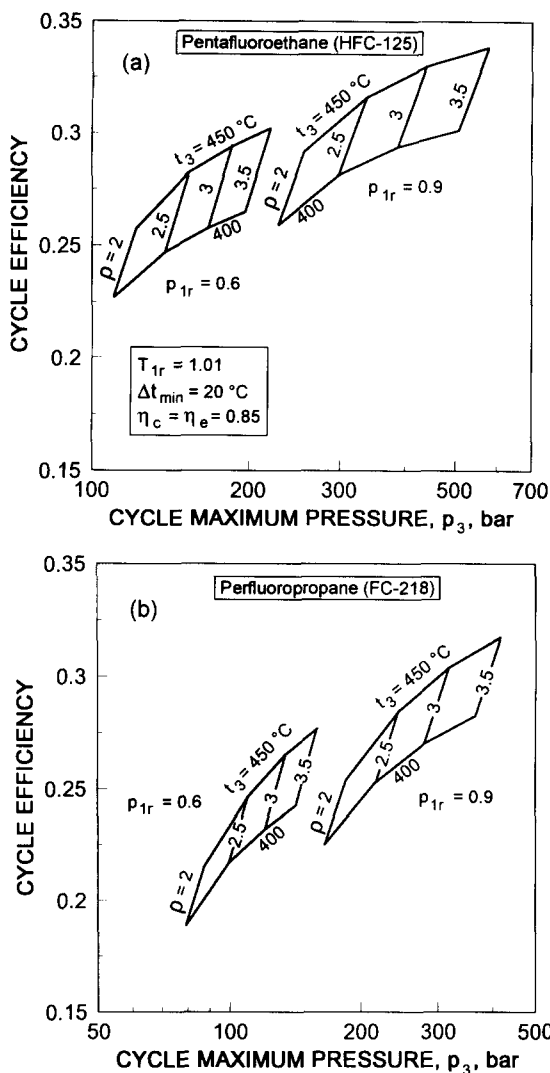


Figure 11 Energy performance of pentafluoroethane and of perfluoropropane real gas Stirling power cycles

Figure 11 Performance énergétique de cycles de Stirling au pentafluoroéthane et au perfluoropropane (gaz réels)

varies as a function of ρ and p_{1r} as reported in Figures 12a and 12b. In the two previous examples the average heat output temperature is 86°C for the C_2HF_5 cycle and 91°C for the C_3F_8 cycle.

The following example shows the energy performance that could be obtained from a Stirling–Stirling system. Let us assume that 100 fuel energy units are used in a HFC125 motor at $t_{max} = 400^\circ\text{C}$ and $\rho = 2.3$. Taking into account 10 units of stack losses, 21.6 work units are generated with an efficiency of 24%, while 68.4 waste heat units are rejected at temperatures in the range 70–110°C. A heat pump using an appropriate fluorocarbon mixture absorbs heat from the ambient at an average temperature of 5°C and produces heat at an average temperature of 60°C with a QF of 0.55 and a COP of 3.78. The engine net work is thus transformed in 81.6 units of heat bringing the overall low-grade heat generation to 150 units delivered to the user heating loop between 45 and 60°C: 45–53.2°C in the heat pump, 53.2–60°C in the prime mover heat exchanger.

A ‘fuel to energy ratio’ of 1.5 is not a remarkably good number. Such a figure is achieved by engine-driven heat pumps of ‘conventional’ design. Also a conventional

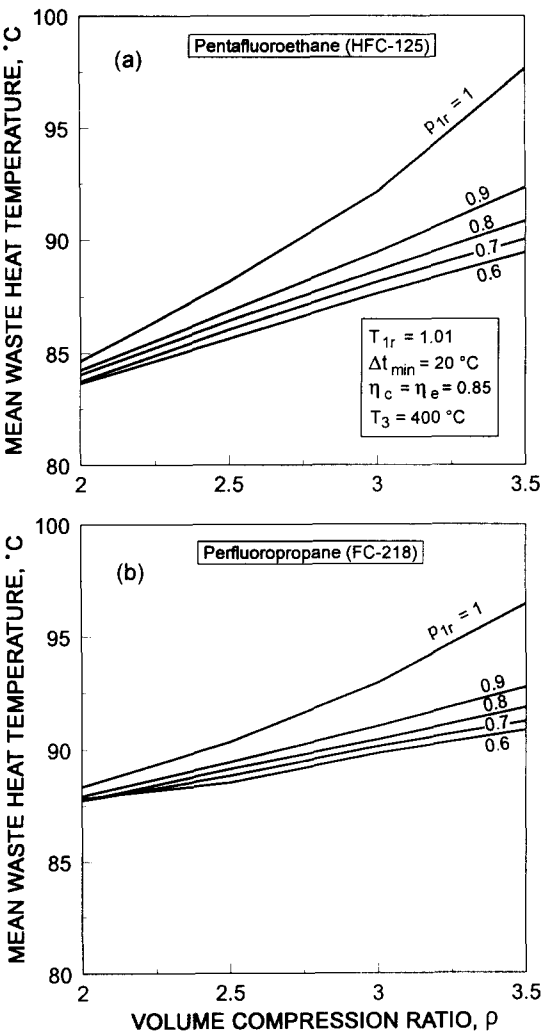


Figure 12 Heat production temperatures of cogenerative pentafluoroethane and perfluoropropane real gas Stirling power cycles

Figure 12 Températures de production de chaleur de cycles de Stirling de cogénération au pentafluoroéthane et au perfluoropropane (gaz réels)

well-designed absorption system gives a COP of 1.5 and a more advanced system (with double effect functions) will give considerably higher values.

Conclusions

The results of the thermodynamic analysis reported in the previous sections support the following conclusions:

- 1. Reversed Stirling heat pump cycles, properly designed within the fluid state diagram so that the expansion process takes place in the vicinity of the critical point where strong real gas effects are present, exhibit a good energy performance with COP much higher than would be achievable with an ideal gas (about twice as much). An overall efficiency similar to that of vapour compression cycles seems possible. High temperature ratios ($\tau = T_{max}/T_{min}$) favour the effectiveness of Stirling real gas cycles, while tending to reduce the efficiency of vapour compression cycles.
- 2. In order to obtain an efficient conversion performance,

a high maximum cycle pressure (around 200 bar) and an effective regeneration of heat are required: Stirling engines are intrinsically capable of standing high pressures and of providing an efficient internal heat transfer.

3. Since in high-quality cycles heat is absorbed from the surroundings slightly above the critical temperature, a strict matching between heat source and fluid properties is required. A variety of zero ODP safe organic fluids are available (pure or as binary mixtures) covering a wide range of critical temperatures. Fluids with a simple molecular structure exhibit the best energy performance and, at the same time, contain the regenerator duty of the heat pump cycle.
4. Since Stirling power cycles can also be designed to benefit from real gas effects, a thermally driven Stirling engine–Stirling heat pump real gas system could be envisaged, producing a good overall fuel energy to heat conversion ratio at top temperatures

around 400–450°C tolerated by some thermodynamically suitable organic fluids.

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